

# Optimal Placement of PMU and RTU by Hybrid Genetic Algorithm and Simulated Annealing for Multiarea Power System State Estimation

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*Abstract*—This paper proposes a hybrid genetic algorithm and simulated annealing (HGS) for solving optimal placement of PMU and RTU for multiarea power system state estimation. Each power system control area includes one PMU and several RTUs. Voltage magnitude, voltage angle, and real and reactive current are measured by PMU while the injection and flow of real and reactive power are measured and monitored through RTU. The power injection and flow measurement pairs are placed to observe the raw data of boundary bus and tie line for data exchange in wide-area state estimator. The critical measurement identification is used to consider the critical measurement free in each area. To reduce the number of measurements and RTUs, a PMU is placed at the bus with the highest number of connected branches. The power injection and flow measurement pairs and RTUs are optimally placed to minimize the installation cost of RTUs and power injection and flow measurement pairs. The results of 10-bus single area, IEEE 14 with 2 areas and 118-bus with 9 areas systems are the optimal measurement placement with critical measurement free. Comparison with simulated annealing (SA) is also made.

*Keywords*— Hybrid genetic algorithm and simulated annealing, Power system state estimation and Measurement placement.

## 1. INTRODUCTION

Power injection and power flow measurement as well as RTU are commonly used in nowadays power system. Conventional power system state estimation uses the online power measurement pairs via RTU for providing the system data to state estimator at control centre. When the system becomes large or connected grids, multiarea power system state estimation should be used to estimate the wide-areas system states. PMU should be introduced into power system for increasing the accuracy of estimated system states. Thus, optimal PMU and RTU placement needs to consider for each area observability with low cost, also entire system states can be estimated by central state estimator.

Multiarea system state estimation by mixed measurements is introduced by many researchers [1-3]. Two levels state estimation are effectively used since the boundary measured data are exchanged [1]. First level, conventional state estimation is introduced to all areas. The voltage phase angle of each area and raw data of boundary buses are sent to the central control centre, then the second level state estimation is implemented for wide-area state estimation. This estimation makes the unbiased estimate for the entire system state [1, 2]. In contrast, the power system is decomposed and then the PMUs are installed to make the area observable [4]. Then, the entire system states of all areas are estimated by the centrally control centre. However, in [4], the tie line data are not observed and also bad data is not mention. More benefit of PMU in power system is voltage stability analysis [5]. The PMU is installed at the bus of each area with largest displacement of the voltage [5]. Similarly, PMU is also installed for a real-time voltage monitoring [6].

In this paper, the optimal placement of a single PMU and RTUs for each power system area is proposed. Each tie line is considered belonging to both this area and neighboring area to ensure that bad data can be identified in the central state estimator. The critical measurement [8] free of each area is considered for bad data detection. A PMU is installed at the bus with the highest incident lines. Number of several RTUs with power injection and flow measurement pairs is minimized by a hybrid genetic algorithm and simulated annealing (HGS) [7]. Also, SA is introduced to solve for the result comparison.

## 2. FUNDAMENTAL OF MEASUREMENT PLACEMENT FOR MULTIAREA STATE ESTIMATION

Area of power system might depend on topography. State estimator of each area estimates the local system states and other applications that gain by local estimated states may be introduced to that area. The central control centre collects the slack bus data of all areas, raw data at boundary buses and tie line, and then the wide-area system states are estimated.

For each area with PMU, the voltage magnitude and angle and real and reactive of current flow in the incident

branches are measured, thus the linear model measurement Jacobian can be written as follows [9]

$$\mathbf{H}_{PMU} = \begin{bmatrix} \delta_i & \delta_j & & & \\ \cdot & \cdot & \cdot & \cdot & \\ \delta_i & \cdot & 1 & \cdot & \cdot \\ \mathbf{I}_{ij} & \cdot & 1 & -1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

If we consider the bus with PMU as a slack bus, the column of  $\delta_i$  will be deleted. Thus, if the conventional measurement pairs, power flow and injection measurement pairs, that connect via RTU, the entire linear model measurement Jacobian or measurement matrix  $\mathbf{H}$  of each area is as follows

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{PMU} \\ \mathbf{H}_{RTU} \end{bmatrix} \quad (1)$$

The  $P\delta$  observability analysis can be introduced. The measurement system is observable if  $rank(\mathbf{H}) = n - 1$ , where  $n$  is number of area system buses. Bad data in measurement of each area can be detected if the measurement system is without critical measurement. Critical measurement ( $cm$ ) can be easily identified by residual analysis [10].

Area of power system for multi-area state estimation can be separated by using the tie line including. This including of tie line makes observable tie line since the measurement will be placed. Also, the areas are overlapping. The typical figure of this separated area is shown as follows

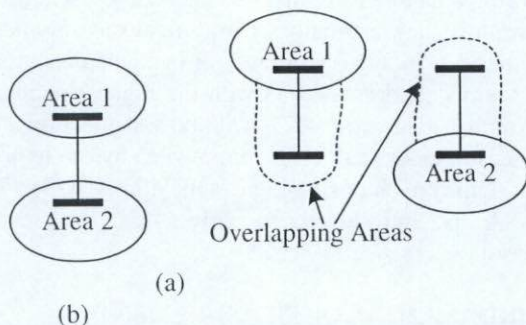


Fig. 1 Typical area separation (a) two areas system (b) area separation for measurement placement

### 3. HGS IMPLEMENTATION FOR OPTIMAL PLACEMENT OF PMU AND RTU

A PMU is considered to place at the bus with maximum branch incident number. Then, HGS [7] is used to minimize the measurement pairs of power injection and flow and RTUs cost. Cost function uses only the conventional measurement cost, since the necessary PMU needs only one per one area. Thus, the

remaining RTUs and measurement pairs cost can be formulated as follows

$$\text{Min } Cost(z) = \sum_{i=1}^{R_T} (CR_i) + \sum_{j=1}^{m_i} CM_{ij} \quad (2)$$

subjects to the observability constraints

$$zero\_pivot = 1 \quad (3)$$

or

$$rank(\mathbf{H}) = N - 1 \quad (4)$$

where  $R_T$  is the number of RTUs,  $m_i$  is the number of measurement pairs (PQ) connected to  $i^{th}$  RTU,  $CR$  is the cost of RTU,  $CM$  is the cost of measurement pair. The matrix  $\mathbf{H}$  in (4) is related with the terms of current flow measurement of PMU and power measurement pair installations. Constraint (3) is used when the triangular factorization or numerical method is used for observability analysis. In (3), zero pivot encounters during the factorization. Constraint (4) is used when the  $P\delta$  observability concept used.

The solution cost evaluation is following to (2) with the penalties. Penalties include the observability and critical measurement. However, the minimum penalty part is observability result.

$$\text{Min } Cost(z) = R_T \cdot CR + m_T \cdot CM + Penalties \quad (5)$$

$$Penalties = Penalty1 + Penalty2$$

$$Penalty1 = [N - 1 - rank(\mathbf{H})](N \cdot N_L)^2 \quad (6)$$

$$Penalty2 = (\text{No. of } cm)(N \cdot N_L)$$

where  $N_L$  is the line numbers,  $m_T$  is the total number of measurement pairs, the measurement pair cost  $CM$  is 4.5 unit of currency and the RTU cost  $CR$  is 100 unit of currency [1, 2]. The first penalty is appeared if system is unobservable. The  $penalty2$  is occurred if the system is with critical measurement.

In fitness evaluation of HGS [7], the measurement matrix  $\mathbf{H}$  is formed for all chromosomes and all generations. The network observability is checked by the  $P\delta$  observability analysis in all chromosomes. The measurement pair and RTU costs will converge to the minimum cost. The fitness function of HGS is according to the cost in (5) as follows.

$$fit = \frac{1}{1 + Cost(z)} \quad (7)$$

The fitness function of HGS is maximized of the inverse of measurement pair and RTU costs and penalty values. The cost plus one protects the divide by zero.

The process of optimal placements of PMU and RTU is start at the system area separation. This separation might be depended on the topography. However, in this paper the system areas are separated as in [1, 3]. The tie line is belonging to both areas. The overall process of optimal placement is as follows.

Step 1: The system is decomposed into each area. System tie line is defined as the radial line of both areas that tie line incident. If the external bus has more than one line incident to same area, only one line is considered for area decomposition.

Step 2: Each area, the system bus with the first largest number of system line incident is selected for PMU placement. This system bus is chosen as the slack bus of area.

Step 3: HGS is introduced for optimal conventional measurement pair and RTU placement of each area.

Here, the current injection measurement [4] is also placed at PMU bus. This current injection measurement handles the critical measurement of current flow measurement in the incident branches, since any single measurement of a PMU can be lost while the branches are observable. This current injection measurement placement condition reduces the critical measurement identifying process of a PMU part.

For described HGS process, the process steps of HGS are explained as follows.

Step 1: Read the system topology of each area.

Step 2: Specify the population size ( $NP$ ), maximum generation limit ( $maxgen$ ) and crossover and mutation probabilities.

Step 3: The  $NP$  chromosome population are randomly initialized.

Step 4: Evaluate the fitness ( $fit$ ) of each initial chromosome using Eq. (7) and find the current best fitness ( $bestfit$ ) and current best chromosome ( $Bchrom$ ) and set the best old fitness ( $Bold$ ) =  $bestfit$ .

Step 5: Set generation counter ( $gen$ ) = 0 and same result counter ( $S$ ) = 0.

Step 6: If  $gen < maxgen$  and  $S < 200$ , set chromosome counter ( $k$ ) = 1. Otherwise, go to Step 7.

Step 6.1: Set the initial current chromosome ( $chrom_{k+1}^{gen+1}$ ) by randomly selecting it from the previous generation.

Step 6.2: If  $k < NP$ ,  
calculate  $\Delta fit = fit(chrom_k^{gen}) - fit(chrom_{k+1}^{gen})$ ,  
set  $T_0 = NP$ . Otherwise, go to Step 6.3.

Step 6.2.1: If  $\Delta fit \geq 0$ , set  $chrom_{k+1}^{gen+1} = chrom_k^{gen}$   
and go to Step 6.2.3.

Step 6.2.2: If  $\exp(\Delta fit / T_p) > \text{random}(0,1)$ ,  
set  $chrom_{k+1}^{gen+1} = chrom_k^{gen}$ ,

where  $T_p = T_0 / k$ . Otherwise,

set  $chrom_{k+1}^{gen+1} = chrom_{k+1}^{gen+1}$ .

Step 6.2.3:  $k = k + 1$ , return to Step 6.2.

Step 6.3: Set the chromosome replacement counter ( $k_1 = 1$ ).

Step 6.4: If  $k_1 \leq \lceil 0.2NP \rceil$ , randomly

generate  $rp \in \{1, \dots, NP\}$ . Otherwise, go to Step 6.5.

Step 6.4.1: Set  $chrom_{rp}^{gen+1} = Bchrom$ .

Step 6.4.2:  $k_1 = k_1 + 1$ , return to Step 6.4.

Step 6.5: Perform the crossover.

Step 6.6: Perform the mutation.

Step 6.7: Evaluate the  $fit$  of each offspring and find the  $bestfit$  and  $Bchrom$ .

Step 6.8: If  $bestfit = Bold$ ,  $S = S + 1$ . Otherwise,  $S = 0$ .

Step 6.9: Set  $Bold = bestfit$ .

Step 6.10: Set  $gen = gen + 1$ , return to Step 6.

Step 7: The  $Bchrom$  is the final solution.

#### 4. NUMERICAL AND PLACEMENT RESULTS

The results of optimal placement of PMU and RTU for multi-areas power system state estimation can be handled the bad data detection in any single measurement, since the measurement system is critical measurement free. These numerical results are show as follows.

Table 1. Numerical results of optimal placements of PMU and RTUs

System	Area	PMU bus Location	No of RTUs		No. of conventional Measurement Pair		Cost	
			HGS	SA	HGS	SA	HGS	SA
10-bus	0	2	3	3	7	7	331.5	331.5
IEEE	1	4	1	1	3	3	113.5	113.5
14-bus	2	6	3	3	7	7	331.5	331.5
IEEE 118-bus	1	12	5	5	10	10	545	545
	2	17	8	8	20	20	890	890
	3	37	5	5	16	16	572	572
	4	32	6	7	20	17	690	776.5
	5	75	5	5	15	15	567.5	567.5
	6	49	7	7	20	20	790	790
	7	100	4	4	10	10	445	445
	8	80	5	5	18	18	581	581
	9	59	6	6	17	17	676.5	676.5

In Table 1, each area with tie line is observable with critical measurement free. Typical placements are shown as follows.

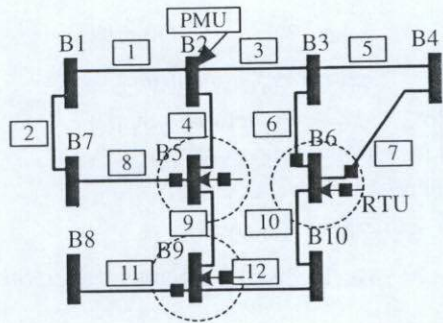


Figure 1. Typical PMU and RTUs placement for 10-bus system with 1 area

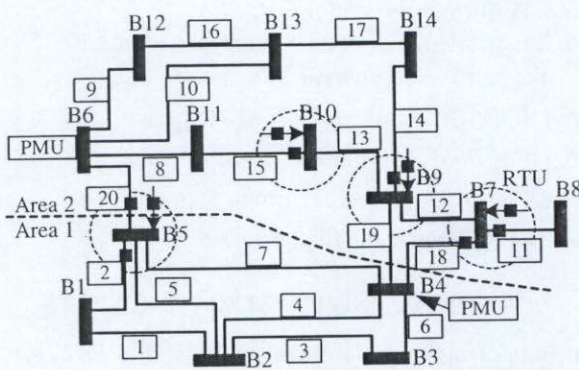


Figure 2. Typical PMUs and RTUs placement for IEEE 14-bus system with 2 areas

### 5. CONCLUSION

Optimal placement of PMU and RTUs is required for multiareas power system state estimation. PMU is placed at the first bus with the maximum number of line incidents. HGS is used to optimize the conventional measurement and RTU placement for area observable with critical measurement free. At the PMU buses, the injection current measurement should be placed to handle the flow current measurement loss of PMU. The measurement placement results are reasonable, since the total number of power flow measurement pair and current measurement are equal to at least as the number of area system buses.

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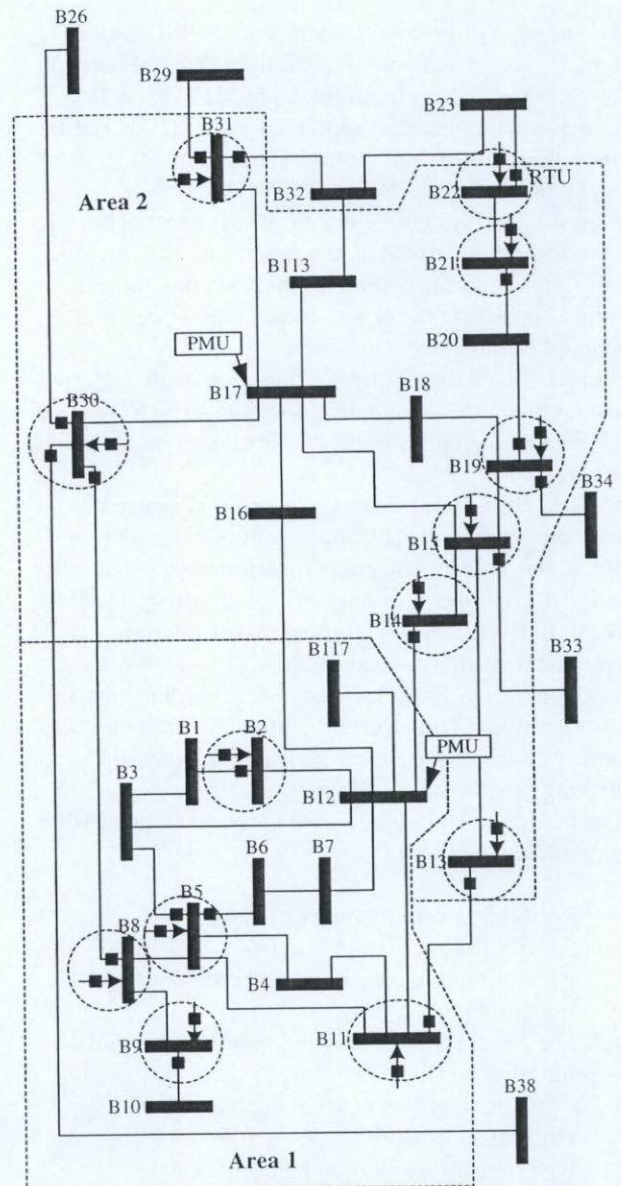


Figure 3. Typical PMUs and RTUs placement for areas 1 and 2 of IEEE 118-bus system

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